Problem 1.
Write the DNF of $p \Leftrightarrow q$ which is defined as $(p \Rightarrow q) \land (q \Rightarrow p)$

Solution.
First we construct the truth table of $(p \Rightarrow q) \land (q \Rightarrow p)$.

p	q	$p \Rightarrow q$	$(q \Rightarrow p)$	$(p \Rightarrow q) \land (q \Rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

The proposition $(p \Rightarrow q) \land (q \Rightarrow p)$ is true precisely when row 1 is true or row 4 is true, which precisely occurs when (p is true AND q is true) OR (p is false AND q is false), so

$$(p \Rightarrow q) \land (q \Rightarrow p) \equiv (p \land q) \lor (\neg p \land \neg q)$$

Problem 2A.
Verify "modus ponen" $(p \Rightarrow q) \land p \Rightarrow q$.

Solution.
To verify $((p \Rightarrow q) \land p \Rightarrow q) \equiv T$, it suffices to show that the logical function $(p \Rightarrow q) \land p \Rightarrow q$ always outputs true in its last column:

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \land p$	$(p \Rightarrow q) \land p \Rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Problem 2B.
Verify "modus tollens" $(p \Rightarrow q) \land \neg q \Rightarrow \neg p$.

Solution.
To verify $((p \Rightarrow q) \land \neg q \Rightarrow \neg p) \equiv T$, it suffices to show that the logical function $(p \Rightarrow q) \land \neg q \Rightarrow \neg p$ always outputs true in its last column:

p	q	$p \Rightarrow q$	$\neg q$	$(p \Rightarrow q) \land \neg q$	$\neg p$	$(p \Rightarrow q) \land \neg q \Rightarrow \neg p$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Problem 3.
Consider the statement "If Bob eats tacos, then he drinks milk."
(a) The converse of this statement is: "If Bob drinks milk, then he eats tacos."
(b) The contrapositive of the original statement is: "If Bob does not drink milk, then Bob does not eat

tacos."

(c) A proposition of the form $p \Rightarrow q$ is false precisely when p is true and q is false; in other words, we must have: Bob does not drink milk, and Bob does eat tacos.

(d) Write the original statement as the form $p \Rightarrow q$ where $p =$ "Bob eats tacos" and $q =$ "Bob drinks milk". Then $p \Rightarrow q$ is true and q is true occurs when $p = T$ or $p = F$ by the truth table below. Therefore we cannot conclude anything about whether Bob eats tacos or not.

p	q	$p \Rightarrow q$
T	**T**	**T**
T	F	F
F	**T**	**T**
F	F	T

(e) Suppose now that the original statement is true and that Bob does not drink milk. Following the notations in (d), we now have $(p \Rightarrow q) = T$ and $q = F$. In the above truth table, this case occurs precisely when $p = F$. In other words, we know that Bob does not eat tacos.

Problem 4.

Let $p =$ "Jon eats pizza" and $q =$ "Jon drinks tea". Then we can formalize the following sentences:

- "Jon eats pizza or drinks tea (possibly both)." as $p \vee q$,
- "If Jon eats pizza, then Jon drinks tea." as $p \Rightarrow q$,
- "If Jon does not eat pizza, then Jon drinks tea." as $\neg p \Rightarrow q$

We write out the truth table involving $p \vee q$, $p \Rightarrow q$, and $\neg q \Rightarrow p$:

p	q	$p \vee q$	$p \Rightarrow q$	$\neg p$	$\neg p \Rightarrow q$
T	T	T	T	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	T	F

We are given that $p \vee q$ is true, so we can eliminate the fourth row. We also know $p \Rightarrow q$ is true, so we eliminate the second row. In the remaining rows (row 1 and row 3), $\neg p \Rightarrow q$ is true. Therefore "If Jon does not eat pizza, then Jon drinks tea" is true.